A conference in honour of Laurent Clozel, LMO, Paris-Saclay

Monday, September 4

9h45-10h15	Harris	*Introduction*
10h15-11h15	Venkatesh	*Some things I learned from Clozel*
11h30-12h30	Thorne	*Level-raising and symmetric power functoriality*
12h30-14h	*Lunch*	
14h-15h	Zhao	*The Langlands parametrization for covering groups*
15h15-16h15	Mézard	*Potentially Barsotti-Tate deformations rings*

Tuesday, September 5

10h15-11h15	Colmez	
11h30-12h30	Burungale	*Zeta elements for elliptic curves and applications*
12h30-14h	*Lunch*	
14h-15h	Beuzart-Plessis	*Isolation of the cuspidal spectrum, multipliers and the real Paley-Wiener theorem*
15h15-16h15	Kret	*The H^0 of Igusa varieties via automorphic forms*
16h30-17h30	Lafforgue	*Progress on local shtukas*
18h-20h	*Cocktail*	

Wednesday, September 6

9h45-10h45	Calegari	*The arithmetic of modular forms*
11h-12h	Lapid	*Some remarks on the geometric side of Arthur's trace formula*
12h15-13h15	Jiang	*Godement-Jacquet Kernels and Clozel Theorem*
13h15-14h30	*Lunch*	

Thursday, September 7

10h15-11h15	Taylor	*The formalism of Shimura varieties*
11h30-12h30	Fouquet	*Celebrating (in) 2023*
12h30-14h	*Lunch*	
14h-15h	Shin	*Endoscopic classification for unitary groups*
15h15-16h15	Schraen	*Multiplicities and (non-)classicality for p-adic automorphic forms*

Friday, September 8

10h15-11h15	Breuil	*Splitting the de Rham complex of the Drinfeld space*
11h30-12h30	Bergeron	*Complex cubic fields and $\mathrm{SL}_3(\mathbb{Z})$-units*
12h30-14h	*Lunch*	
14h-15h	Chenevier	*On level one algebraic cuspforms of $\mathrm{GL}(n)$ over $\mathbb{Q}$*
15h15-16h15	Emerton	

Abstracts :

— **Bergeron** : *Complex cubic fields and* $\mathrm{SL}_3(\mathbb{Z})$*-units*

In the early 2000's Felder and Varchenko have defined a generalisation of the Euler gamma function that is associated to an elliptic curve. This remarkable *elliptic gamma function* is meromorphic in several variables and comes from mathematical physic. It satisfies modular functional equations under the group $\mathrm{SL}_3(\mathbb{Z})$ which make it an analogue of the Jacobi theta function. In this paper, we unveil the place that this function and its avatars play in number theory. Our main thesis is that these functions play the role of meromorphic modular functions in extending the theory of complex multiplication to complex cubic fields. In other words we propose a conjectural solution to Hilbert's 12th problem for complex cubic fields. We give a lot of numerical evidences that support this conjecture. In addition we prove an analogue of the Kronecker limit formula that expresses the logarithm of the modulus of these conjectural units as the derivative at $s = 0$ of a partial zeta function. This relates our conjectural units to the conjectural Stark units. This is a joint work with Pierre Charollois and Luis Garcia.

— **Beuzart-Plessis** : *Isolation of the cuspidal spectrum, multipliers and the real Paley-Wiener theorem*

In this talk, I will review a construction of convolution operators that isolate certain cuspidal representations from the rest of the automorphic spectrum. This allows for drastic simplifications in the analysis of spectral sides of certain (relative) trace formulas. For this, we combine the action of spherical Hecke algebras at unramified places with that of an algebra of "multipliers" at Archimedean places. In particular, it is crucial that the multiplier algebra we use be sufficiently large. Time permitting, I will also speculate on a possible extension of Arthur's Paley-Wiener theorem for real reductive groups that fits very naturally in this story. This is based on joint work with Yifeng Liu, Wei Zhang and Xinwen Zhu.

— **Burungale** : *Zeta elements for elliptic curves and applications*

The talk plans to outline the existence of two-variable zeta element over an imaginary quadratic field for an elliptic curve defined over $\mathbb{Q}$. Its arithmetic consequences include proof of Kobayashi's main conjecture for semistable curves. (Joint with C. Skinner, Y. Tian and X. Wan.)

— **Calegari** : *The arithmetic of modular forms*

We discuss a number of new applications of transcendental number theory to the theory of modular forms.

— **Fouquet** : *Celebrating (in) 2023*

There are great mathematicians, and there are small mathematicians. Laurent Clozel is a great mathematician. This talk, as befits the speaker, will be very small : it will be concerned mostly about one elliptic curve at one prime - the only rational elliptic curve E of conductor 2023 and algebraic rank 2, and the prime 17. I will show that the Tate-Shafarevich group of this elliptic curve at this prime has the order predicted by the Birch and Swinnerton-Dyer Conjecture (note that E has rank 2 and very bad reduction at 17 so that the result is small, but nonetheless new). Along the way, we will encounter universal deformation rings, p-adic families of automorphic representations, special values of L-functions and many other things we could not possibly have studied without Laurent Clozel.

— **Jiang** : *Godement-Jacquet Kernels and Clozel Theorem*

In a recent work of L. Clozel, certain kernel functions on GL(1) and its connection with problems in analytic number theory were introduced and studied. In

this talk, I will explain that the reformulation of the work of Godement-Jacquet can be used to understand the framework of those kernels and its dual kernels for any irreducible cuspidal automorphic representations π of GL(n). Along the way, I will also discuss other related problems. This is based on my joint work with Zhilin Luo and Zhalin Li.

— **Lafforgue** : *Progress on local shtukas*

I will explain recent progress on local shtukas in characteristic p, in a joint work with Genestier and a work in progress with Eteve, Gaitsgory and Genestier. I will explain also the compatibility with the cohomology of global shtukas.

— **Lapid** : *Some remarks on the geometric side of Arthur's trace formula*

In this talk I will report on work in progress with Tobias Finis aiming at expressing the unipotent contribution of Arthur's trace formula in terms of zeta functions of prehomogeneous vector spaces.

— **Mézard** : *Potentially Barsotti-Tate deformations rings*

In collaboration with B. Le Hung and S. Morra, we develop a local model theory for moduli stacks of étale (φ, Γ)-modules corresponding to 2-dimensional non-scalar tame potentially Barsotti–Tate Galois representations of the Galois group of an unramified extension of $\mathbb{Q}_p$. We derive from this explicit presentations of potentially Barsotti–Tate deformation rings, in particular the proof of the various Caruso–David–Mézard conjectures.

— **Schraen** : *Multiplicities and (non-)classicality for p-adic automorphic forms*

We prove the existence of non-classical overconvergent p-adic automorphic forms for the group U(3) with a classical system of eigenvalues. I will explain how this is related to the existence of multiplicities for locally algebraic vectors in the expected p-adic Langlands correspondence and a relation with a geometric form of this correspondence. This is a joint work with Eugen Hellmann and Valentin Hernandez.

— **Shin** : *Endoscopic classification for unitary groups*

Arthur proved the endoscopic classification theorem for automorphic representations of quasi-split symplectic and orthogonal groups about a decade ago. It was extended to quasi-split unitary groups by Mok, and partly to non-quasi-split unitary groups by Kaletha, Minguez, White, and myself. In particular, this strengthens the cohomological base change for unitary groups due to Clozel and Labesse. All of these theorems, except Clozel-Labesse's base change, are conditional on certain expected results. I will discuss their current status as well as a related ongoing project with Atobe, Gan, Ichino, Kaletha, and Minguez.

— **Taylor** : *The formalism of Shimura varieties*

This is joint work with Jack Sempliner. In the 1970's Deligne proposed a formalism for Shimura varieties and Langlands conjectured a formula for the action of Galois on them. Deligne's formalism involves 'Shimura data' which parametrizes a Shimura together with a preferred embedding of its field of definition into the complex numbers. We propose a different formalism that directly parametrizes a Shimura variety over any field of characteristic 0. To this end we have to revisit the theory of conjugation of Shimura varieties conjectured by Langlands and established by Milne and others. We hope that our reformulation of this work also helps to clarify the theory. A key tool is Kottwitz's definition of the groups B(G).

In this talk I will sketch the formalism of Deligne and Langlands and point out what we see as its shortcomings. I will then introduce Kottwitz's cohomology theory for extensions of Galois groups, and explain how to make cocycles a canonical object, not just cohomology classes. Finally I shall explain a reformulation of

the theory of conjugation of Deligne's Shimura varieties and propose an alternative formalism for Shimura varieties.

— **Thorne** : *Level-raising and symmetric power functoriality*

In 2014 Clozel and I formulated a programme to prove the existence of all symmetric power liftings of Hilbert modular forms of regular weight, conditional on conjectures concerning the existence of level-raising congruences and cases of Langlands tensor product functoriality. In 2022 I proved the existence of sufficiently many of these level-raising congruences and, in joint work with James Newton, applied this to carry out our programme unconditionally. In this talk I will review the context and discuss the proof of existence of these congruences.

— **Venkatesh** : *Some things I learned from Clozel*

In 2002, shortly after I finished my PhD, I attended a series of lectures given by Clozel at Park City. There he made a beautiful observation to which I have returned many times over my career, namely, Arthur's global conjectures have very surprising "purity" implications for questions in local harmonic analysis.

This implies, in particular, that many features of the harmonic analysis of G acting on $L^2(G/H)$ (for G a reductive group over a local field) are controlled by a single nilpotent orbit in the dual group $\hat{G}$. I will sketch Clozel's argument and why I find this phenomenon so interesting. I will then discuss the important role of this orbit in "relative Langlands duality".

— **Zhao** : *The Langlands parametrization for covering groups*

In this talk, I will report on some recent progress in the Langlands program for covering groups, as initiated by Weissman, Gan, and Gao. This extension of the Langlands program puts certain topological covers of reductive groups on equal footing as linear algebraic groups. I will explain the cohomological origin of these covers and how it allows us to transport known constructions of the Langlands parametrization to covering groups.